The Effectiveness of the Kozai Mechanism in the Galactic Centre

Philip Chang[1]⋆
[1] Department of Astronomy, 601 Campbell Hall, University of California, Berkeley, CA 94720

5 November 2008

ABSTRACT
I examine the effectiveness of Kozai oscillations in the centres of galaxies and in particular the Galactic centre using standard techniques from celestial mechanics. In particular, I study the effects of a stellar bulge potential and general relativity on Kozai oscillations, which are induced by stellar discs. Löckmann et al. (2008) recently suggested that Kozai oscillations induced by the two young massive stellar discs in the Galactic centre drives the orbits of the young stars to large eccentricity ($e \approx 1$). If some of these young eccentric stars are in binaries, they would be disrupted near pericentre, leaving one star in a tight orbit around the central SMBH and producing the S-star population. I find that the *spherical* stellar bulge suppresses Kozai oscillations, when its enclosed mass inside of a test body is of order the mass in the stellar disc(s). Since the stellar bulge in the Galactic centre is much larger than the stellar discs, Kozai oscillations *due to the stellar discs* are likely suppressed. Whether Kozai oscillations are induced from other nonspherical components to the potential (for instance, a flattened stellar bulge) is yet to be determined.

Key words: Galaxy: centre – celestial mechanics

1 INTRODUCTION

Within 0.04 pc from Sgr A*, a swarm of young stars, known as the S-stars(Eckart & Genzel 1997) or S0-stars (Ghez et al. 2005), orbit the central supermassive black hole (SMBH) in highly eccentric Keplerian orbits with random inclination, i.e. an isotropic distribution. These S-stars are typically of type B2 with a mass of $\sim 10 M_\odot$ and a main sequence lifetime ~ 20 Myrs (Alexander 2005). Outside of these S-stars (0.04 pc to 0.4 pc), another population of young, massive stars, which consist of $\sim 6 \pm 1$ Myr O- and early B-type stars, have a very different distribution. Rather than being isotropic, these stars are organized into one (Lu et al. 2006) or two disc(s) (Paumard et al. 2006).

The exact number of discs remains controversial. Paumard et al. (2006) find that these young stars are arranged in two stellar discs, which are inclined at $\approx$ 115 degrees relative to one another (Paumard et al. 2006; Levin & Beloborodov 2003; Genzel et al. 2003). The stars in one disc (which is the more massive disc) have roughly circular orbits with the typical eccentricity < 0.4, while the stars in the other less massive disc are very eccentric ($e > 0.6$). On the other hand Lu et al. (2006) find only one disc of stars, the more massive disc which Paumard et al. (2006) iden-

tifies, and a "halo" population of young stars in random, highly inclined, and eccentric orbits.

The origin of the young stars in the central parsec of the Galaxy remains an unsolved problem (Alexander 2005). For the young stars between 0.04 and 0.4 pc, their arrangement in a disc suggests that they may have formed in-situ from the condensation of a gas disc (Levin & Beloborodov 2003; Nayakshin & Cuadra 2005; Nayakshin, Cuadra & Springel 2007; Levin 2007). Such star formation would be expected in accretion discs at large radii due to the fragmentation from self-gravity (Paczynski 1978; Kolychalov & Syunyaev 1980; Goodman 2003; Thompson, Quataert & Murray 2005; Chang 2008). The more massive, more securely identified disc seem to favor this in-situ model as the stellar orbits are circular and dynamically cold (Paumard et al. 2006; Lu et al. 2006). Alternatively, these stars may have formed in a large cluster at large radii, which spiraled inward due to dynamical friction (Gerhard 2001; Hansen & Milosavljević 2003; Berukoff & Hansen 2006).

The problem of the S-stars' formation is even more daunting than that of the young, massive stars between 0.04 and 0.4 pc (for a review see Alexander 2005). For the S-stars, the tidal field from the SMBH and their current random orientation argues against both an in-situ disc formation scenario and a sinking cluster (Ghez et al. 2005). The S stars may have formed via a different channel than the young, massive stars. Levin (2007) has argued that

arXiv:0811.0829v1 [astro-ph] 5 Nov 2008

type 1/2 migration to small radii (Goldreich & Tremaine 1978), followed by resonant relaxation (Rauch & Tremaine 1996) is a viable formation channel. On the other hand, Perets, Hopman & Alexander (2007) argues that the large population of massive perturbers near the Galactic centre (GC) increases the relaxation rate to such an extent that these S-stars may have been formed when binaries, which are scattered into large eccentricities, are disrupted near pericentre (Hills 1988; Gould & Quillen 2003).

Recently, a rather elegant scenario has been proposed by (Löckmann, Baumgardt & Kroupa 2008, hereafter LBK) for the origin of the S stars and their link to the young, massive stars which surround them. LBK showed that the S-stars could be formed from tidal disruption of binaries (Hills 1988; Gould & Quillen 2003) which are driven to large eccentricity from Kozai oscillations (Kozai 1962; Lidov 1962) induced from the outer two young massive stellar discs. Earlier work by Subr, Karas and collaborators (Šubr et al. 2004; Šubr & Karas 2005; Karas & Šubr 2007) studied the case for a single disc (with gasdynamic dissipative effects for the case of a fossil gas disc). This is indeed a very attractive proposal as it naturally would explain their isotropic distribution, large eccentricity, and apparent youth. Using an N-body calculation with relativistic correction of up to 2.5 post-Newtonian orders (Löckmann & Baumgardt 2008), they showed that stars (or binaries) from the two discs can achieve eccentricities as large as $e \approx 0.999$. Relativistic corrections, which typically, can damp large eccentricities in the Kozai mechanism (Holman, Touma & Tremaine 1997; Fabrycky & Tremaine 2007) appear not to be important (LBK). Binaries in these eccentric orbits that are disrupted near pericentre will result in one of the stars in a tight orbit around the SMBH. The reduction of semimajor axis from such a binary fission event would be of order a factor of ten (Gould & Quillen 2003) and would be a viable mechanism for the formation of the S-stars.

The beauty of LBK's scenario is that it ties in observed properties of the S-star distribution, namely their isotropy, eccentricity, and small semimajor axis with the observed properties of the young stellar discs, namely their high inclination of 115 degrees relative to each other. However, LBK's result is surprising as the Kozai mechanism is fairly delicate and can be greatly suppressed if additional perturbations to the gravitational potential such as a stellar cusp are included. Thus, I am motivated to examine of the basic physics of this scenario and ask the question: to what degree might Kozai oscillations be important in the centres of galaxies. In this paper, I study the nature of the Kozai mechanism central this scenario. I present my basic model and the basic equations for the Keplerian orbital parameters in §2. In §3, I apply my basic model to the Galactic Centre. I numerically compute the evolution of a star subject to the perturbed potential from a single disc, a single disc with a stellar bulge, two discs, and two discs with a bulge. I show that including the spherical stellar distribution of sufficient mass suppresses the Kozai mechanism. I then argue that the spherical stellar distribution in Galactic centre is more than adequate of this purpose. I discuss some of these implications and conclude in §4.

2 SECULAR EVOLUTION

I now describe the model problem central to this study. Two massive stellar discs with mass M_1 and M_2 orbit a central SMBH with mass $M_0 \gg M_1, M_2$ at an inclination relative to one another of i_0. The surface density of these two stellar discs, $\Sigma_{1,2}$, is assumed to be a power law with index $-p$, i.e., $\Sigma_{1,2} = \Sigma_{1,2,0}(r/r_0)^{-p}$, where r is the radial coordinate, r_0 is a reference radius, and $\Sigma_{1,2,0}$ is the normalization for disc 1 and 2 respectively. Without loss of generality, I orient my axis such that the reference plane is in the plane of the M_1 disc and I presume that $M_1 \geqslant M_2$. Also surrounding the SMBH is a spherical stellar power law distribution with index $-q$, $n_* = n_0(r/r_0)^{-q}$. A test body orbits the SMBH in a near-Keplerian orbit with semimajor axis, a, inclination, i, and eccentricity, e.

I first consider the case of a perturbing mass, δm, in a circular orbit of radius, r, around the central mass and its effect on the test body. I will work exclusively in the secular approximation. This problem is well studied by many authors (see for instance Innanen et al. 1997; Kiseleva et al. 1998; Ford et al. 2000). In particular, Eggleton et al. (1998) (see also Eggleton & Kiseleva-Eggleton 2001; Fabrycky & Tremaine 2007) has introduced a nice formalism which I find very flexible, powerful, and especially useful for studying multiple perturbing bodies which are at large inclinations relative to one another. Using the notation of Eggleton & Kiseleva-Eggleton (2001) and Fabrycky & Tremaine (2007) and dropping dissipative and tidal terms, the governing equations are:

$$\frac{1}{e}\frac{d\vec{e}}{dt} = (Z_{\rm GR} + Z_*)\,\hat{\boldsymbol{q}}$$
$$- \left(1 - e^2\right)\left[5S_{eq}\hat{\boldsymbol{e}} - (4S_{ee} - S_{qq})\,\hat{\boldsymbol{q}} + S_{qh}\hat{\boldsymbol{h}}\right], \quad (1)$$

$$\frac{1}{h}\frac{d\vec{h}}{dt} = \left(1 - e^2\right)S_{qh}\hat{\boldsymbol{e}} - \left(4e^2 + 1\right)S_{eh}\hat{\boldsymbol{q}} + 5e^2 S_{eq}\hat{\boldsymbol{h}}, \quad (2)$$

where $\vec{e}$ is the Laplace-Runge-Lenz vector, whose magnitude is the eccentricity, e, $\vec{h}$ is the reduced orbital angular momentum vector, $\hat{\boldsymbol{h}}$ and $\hat{\boldsymbol{e}}$ are the normalized vectors in the direction of $\vec{h}$ and $\vec{e}$ respectively, and $\hat{\boldsymbol{q}} = \hat{\boldsymbol{h}} \times \hat{\boldsymbol{e}}$ is the normal vector that completes the triad. $(\hat{\boldsymbol{h}}, \hat{\boldsymbol{e}}, \hat{\boldsymbol{q}})$. The disturbing tensor[1] is $S_{xy} = C\left[\delta_{xy} - 3(\hat{\boldsymbol{h}}' \cdot \hat{\boldsymbol{x}})(\hat{\boldsymbol{h}}' \cdot \hat{\boldsymbol{y}})\right]$, where $\hat{\boldsymbol{h}}'$ is the normalized angular momentum vectors of the perturbing mass and the constant, C, is

$$C = n(a)\frac{\delta m}{M_0}\left(\frac{a}{r}\right)^3 \left(1 - e^2\right)^{-1/2}, \quad (3)$$

where $n(a) = \sqrt{GM_0/a^3}$ is the mean motion. Finally $Z_{\rm GR}$ and Z_* represents the apsidal motion from the effects of general relativity and the stellar cusp. The precession term due to general relativity is (Eggleton & Kiseleva-Eggleton 2001; Fabrycky & Tremaine 2007):

$$Z_{\rm GR} = \frac{3}{2}\frac{r_g}{a}\frac{n}{1 - e^2}, \quad (4)$$

where $r_g = 2GM_0/c^2$ is the gravitational radius. The precession due to the stellar bulge potential is (Ivanov et al. 2005, see their eq.(14) and (15) and appendix A)

---

[1] I am unaware of a name for this tensor, so for the sake of nomenclature, I have chosen to call this the disturbing tensor.

$$Z_* = -\kappa n \frac{M_*(a)}{M_0}, \tag{5}$$

where $M_*(a) = 4\pi \int_0^a m_* n(r) r^2 dr$ is the mass of the bulge stars inside of sphere whose radius is equal to the test body's semimajor axis, a,

$$\kappa = \frac{\Gamma(5/2 - q)}{\sqrt{\pi}\Gamma(3 - q)} \tag{6}$$

is a constant of order unity, and Γ is the gamma function.

I now calculate the effect of a disc. Taking $\delta m \to dm = 2\pi \Sigma r dr$, I find that the effect of a disc is a modification of equation (3) to be

$$C_{\rm d} = n(a) M_0^{-1} a^3 \left(1 - e^2\right)^{-1/2} 2\pi \int_{r_{\rm in}}^{r_{\rm out}} dr \Sigma r^{-2} dr, \tag{7}$$

where $C_{\rm d}$ is the constant associated with the disc, $r_{\rm in}$ is the disc inner radius, and $r_{\rm out}$ is the outer radius of the disc. For $r_{\rm out} \gg r_{\rm in}$, this gives

$$C_{\rm d} = n(a) \frac{M_{\rm eff}}{M_0} \left(\frac{a}{r_{\rm in}}\right)^3 \left(1 - e^2\right)^{-1/2} \tag{8}$$

where $M_{\rm eff} = 2/(1 + p)\pi \Sigma_{1,0} r_0^2$ is the effective mass of the disc, and $M_{\rm d} = 2\pi \int_{r_{\rm in}}^{r_{\rm out}} dr r \Sigma$ is the mass of the disc. Note that the effect of a massive extended stellar disc with $r_{\rm out} \gg r_{\rm in}$ can be reduced to a single ring with mass $M_{\rm eff}$ at a radius of $r_{1,2{\rm in}}$ for $p > -1$. For $p = -1$, the contribution is logarithmic in radius and for $p < -1$, the outer radius is more relevant.

The use of the governing equations (1) and (2) is especially advantageous for studying the secular effects of multiple perturbing rings. Like the Lagrange-Laplace planetary equations (Brouwer & Clemence 1961; Murray & Dermott 2000), the governing equations (1) and (2) are linear. Hence, the effect of additional perturbing masses is just a matter of adding their associated disturbing function (in the case of the Lagrange-Laplace planetary equations) or the perturbing potential, S_{xy}, in the case of equations (1) and (2). The calculation of the *inclined* disturbing function in the case of the secularly averaged Laplace-Langrange's planetary equations (Brouwer & Clemence 1961; Innanen et al. 1997; Kiseleva et al. 1998; Murray & Dermott 2000; Ivanov et al. 2005) is rather involved and I have found it to be numerically difficult to solve. By contrast including additional masses is trivial for the governing equations (1) and (2). For a number of perturbing masses, δm_i, the disturbing tensor, S_{xy}, is a sum over all disturbing tensors from all the masses, or

$$S_{xy} = \sum_i C_i \left[\delta_{xy} - 3(\hat{\boldsymbol{h}}_i' \cdot \hat{\boldsymbol{x}})(\hat{\boldsymbol{h}}_i' \cdot \hat{\boldsymbol{y}})\right], \tag{9}$$

where $\hat{\boldsymbol{h}}_i'$ is the normalized angular momentum vectors of the perturbing mass δm_i. The associated constant, C_i, is

$$C_i = n(a) \frac{\delta m_i}{M_0} \left(\frac{a}{r_i}\right)^3 \left(1 - e^2\right)^{-1/2}, \tag{10}$$

where r_i is the radial position of the perturbing mass.

3   APPLICATION TO THE GALACTIC CENTRE

In the case of the Galactic centre, (Paumard et al. 2006) find that the stellar distribution follows $p \approx 2$, $r_{1,2,{\rm in}} = 0.1$
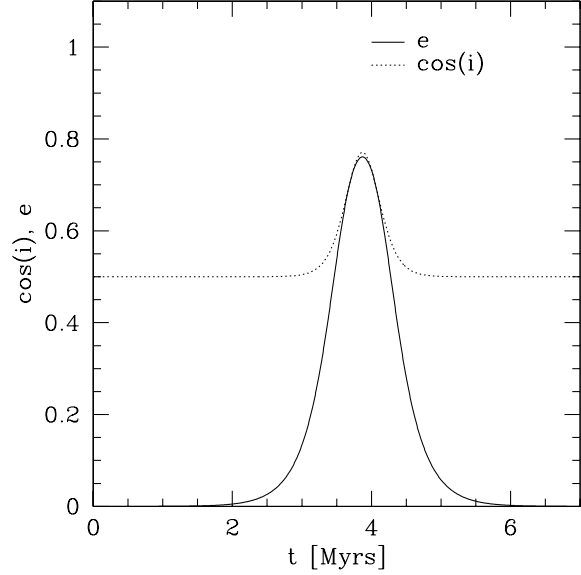

Figure 1. Simple illustration of the Kozai oscillation for one disc of mass, $M_{\rm eff} = 2 \times 10^3 {\rm M}_\odot$ for a test particle (star) with an initial inclination of 60 degrees relative to the plane of the disc. The test particle's eccentricity and inclination are shown as a solid and dotted lines respectively. The precession from general relativity and the stellar bulge potential are ignored in this case.

pc, and $r_{1,2,{\rm out}} = 0.5$ pc for the two disc, which are oriented at 115 ± 7 degrees with respect to one another. The upper limit for the mass of the clockwise disc is $M_{\rm d,1} < 10^4 {\rm M}_\odot$, and the upper limit for the mass of the counterclockwise disc is $M_{\rm d,2} < 5 \times 10^3 {\rm M}_\odot$ (Paumard et al. 2006), which is consistent with Nayakshin et al. (2006)'s dynamical limits of $\lesssim 10^4 {\rm M}_\odot$ for the masses of two discs. For a top-heavy initial mass function (IMF), Paumard et al. (2006) derives a lower limit for the stellar mass of the two discs to be $3500 {\rm M}_\odot$ and $1400 {\rm M}_\odot$. Hence for a range in disc masses between $M_{\rm d,1} = 3500 - 10^4 {\rm M}_\odot$ and $M_{\rm d,2} = 1400 - 5000 {\rm M}_\odot$, the effective disc masses are $M_{1,\rm eff} \approx 0.6 - 2 \times 10^3 {\rm M}_\odot$ and $M_{2,\rm eff} \approx 0.3 - 1 \times 10^3 {\rm M}_\odot$.

I numerically calculate the evolution of equation (1) and (2), using a standard Runge-Kutta algorithm (Press et al. 1992). As an illustration, I plot the eccentricity (solid line) and inclination (dotted line) for a test particle that is in orbit around a SMBH and is initially inclined relative at 60 degrees to a single massive disc with $M_{\rm eff} = 2000 {\rm M}_\odot$ in Figure 1. For this case, I have taken $Z_{\rm GR} = 0$ and $Z_* = 0$. Note the characteristic behavior of the Kozai oscillation, where the inclination and eccentricity vary in phase and the initial inclination determines the amplitude of the oscillation due to the conservation of the Kozai integral or z-component of the angular momentum vector, $L_{\rm z} = (1 - e^2) \cos^2 i$ (Kozai 1962).

The axissymmetry, but not spherical symmetry, of the potential conserves $L_{\rm z}$, but not the total angular momentum vector, L. Thus, an initially inclined low eccentricity orbit achieves very high eccentricities. For the case of the region around a SMBH, Šubr & Karas (2005) and Karas & Šubr (2007) studied the effect of fossil gas disc on the orbits of a nuclear star cluster. In the case of the a single disc,

Šubr & Karas (2005) and Karas & Šubr (2007) showed that there are two kinds of orbits: orbits which librate around $\omega = \pi/2$ and $3\pi/2$ and orbits which span over $\omega = [0, 2\pi]$, where ω is the argument of pericentre. In the particular case of a fossil gas disc, dissipative interactions from star-crossings of the fossil gas disc dissipates energy, resulting in a slow decay of the semi-major axis (Karas & Šubr 2007).

I now study the effect of Kozai oscillations including the effects of general relativity, Z_{GR} and the second disc. In agreement with the LBK's claim, I do not find the effect of relativity to be significant for eccentricities up to 0.999.

I now include the effect of the second disc, which is less massive at $\approx 1.4 - 5 \times 10^3 \mathrm{M}_\odot$ than the first disc ($3.5 - 10 \times 10^3 \mathrm{M}_\odot$). The observed inclination is fairly narrow 115 ± 7 degrees, so I choose to fix the inclination at 115 degrees. In Figure 2, I show the maximum eccentricity, $e_{\max}$, reached as a function of initial $\cos i$ relative to the reference plane (which is the plane of the disc for the one disc case and the plane of the more massive disc in the two disc case). For two discs, $e_{\max}$ as a function of $\cos i$ is considerably more complex than the case for one disc. Note that for low mass discs (either for one disc or two disc case), there is insufficient time (7 Myrs) for stars to reach high eccentricity. For two discs, the range of initial inclinations that generate large eccentricities is larger compared to the single disc. Also note, that large eccentricities can be reached for two discs in the neighborhood around the inclination of the second disc. The addition of a second disc increases the available inclinations over which test bodies, i.e., stars, can reach large eccentricities. Note also that even in the most optimistic scenario, stars do not oscillate to high eccentricity for $\cos i \approx \pm 1$. Stars that are close the to more massive stellar disc do not reach large inclination in spite of the second disc. Hence, stars and binaries that are driven to large eccentricities must come from the less massive disc, which supports LBK's result that stars in the CCW disc (the less massive disc) are more likely to be driven to large eccentricity.

The axissymmetry of a single disc conserves L_z. As a result, a polar plot of e-ω (see for instance Šubr & Karas 2005; Karas & Šubr 2007) of an orbit generates a closed curve. On the other hand, when a second disc is included, axissymmetry is broken and an orbit no longer generates close curves in a polar e-ω plot.

When the stellar bulge potential is included, Z_*, the dynamics changes completely. For the bulge potential I take $q = 1.4$ (which gives $\kappa \approx 0.6$ for eq.[5]) and $n_0 = \rho_0/\mathrm{M}_\odot$ is the number density of stars, where ρ_0 is the mass density of stars (Genzel et al. 2003; Yu et al. 2007). I plot its effect as a function of $M_*(a = 0.1\,\mathrm{pc})$ in Figure 3 for the most optimistic scenario $M_{\mathrm{d},1} = 10^4 \mathrm{M}_\odot$ and $M_{\mathrm{d},2} = 5000\mathrm{M}_\odot$. As this figure shows, sufficiently large stellar bulges ($M_*(a = 0.1\,\mathrm{pc}) \gtrsim 3500\mathrm{M}_\odot$ for one disc and ($M_*(a = 0.1\,\mathrm{pc}) \gtrsim 4500\mathrm{M}_\odot$ for two discs) suppresses the Kozai mechanism.

The mass found that is needed to suppress Kozai oscillations agrees well with simple analytic arguments. The Kozai mechanism is known to be suppressed by non-Keplerian contributions to the potential such as those introduced by general relativity or additional masses in the system (see the case for multiple planets, Tremaine & Zakamska 2004). Typically if the period for apsidal precession is of order or shorter than the period of the Kozai oscillations, then the
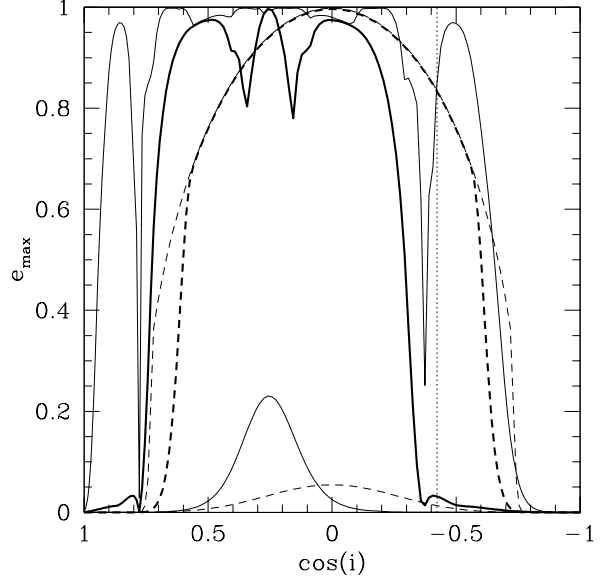

Figure 2. Maximum eccentricity as a function of initial inclination for one disc (dashed lines) with $M_{1,\mathrm{eff}} = 0.6$ (lower dashed line), 1.2 (middle thick dashed line), $2 \times 10^3 \mathrm{M}_\odot$ (upper dashed line) and two discs with $M_{1,\mathrm{eff}} = 0.6$ (lower solid line), 1.2 (middle thick solid line), $2 \times 10^3 \mathrm{M}_\odot$ (upper solid line) with respective $M_{2,\mathrm{eff}} = 0.3, 0.6, 1 \times 10^3 \mathrm{M}_\odot$). As a function of phase space, note that the regions available for large eccentricities ($e \gtrsim 0.95$) is much larger for two discs as opposed to one disc. The vertical dotted line indicates the position of the second disc relative to the first for the parameters of the two discs in the Galactic centre, i.e., an inclination of 115 degrees.

Kozai oscillations are suppressed (Tremaine & Zakamska 2004; Fabrycky & Tremaine 2007). The period of the Kozai oscillation is of order (from studying eq.[3]):

$$\tau_{\mathrm{K}}^{-1} \sim \zeta_{1,2} \sim n \left(\frac{a}{r_{1,2,\mathrm{in}}}\right)^3 \left(\frac{M_{\mathrm{eff}}}{M_0}\right). \tag{11}$$

whereas the timescale for apsidal precession due to the stellar bulge is of order (from studying eq.[5])

$$\tau_*^{-1} \sim n \frac{M_*(a)}{M_0}. \tag{12}$$

Hence, the requirement that the apsidal precession period be longer than the period for Kozai oscillations imply $M_* \lesssim M_{\mathrm{eff}}$, matching the expectations from the more detailed calculation in Figure 3.

I now argue that the mass of stellar bulge is much larger than what is needed to suppress the Kozai mechanism. From observations, the measured $M_*(a = 0.1\,\mathrm{pc})$ is $\approx 6 \times 10^4 \mathrm{M}_\odot$, using the values for the central stellar density from Genzel et al. (2003) (see also Schödel et al. 2007). The observed mass is over an order of magnitude larger than what is needed to suppress the Kozai mechanism. It is significantly larger than the mass of the two young stellar discs and, therefore, Kozai oscillations induced by the two young massive stellar discs are likely suppressed.

The suppression of Kozai oscillations due to a spherical distribution of stars is well known for the case of a single disc (Ivanov et al. 2005; Karas & Šubr 2007). Ivanov et al. (2005) argues that in the case of a single ring, Kozai oscilla-
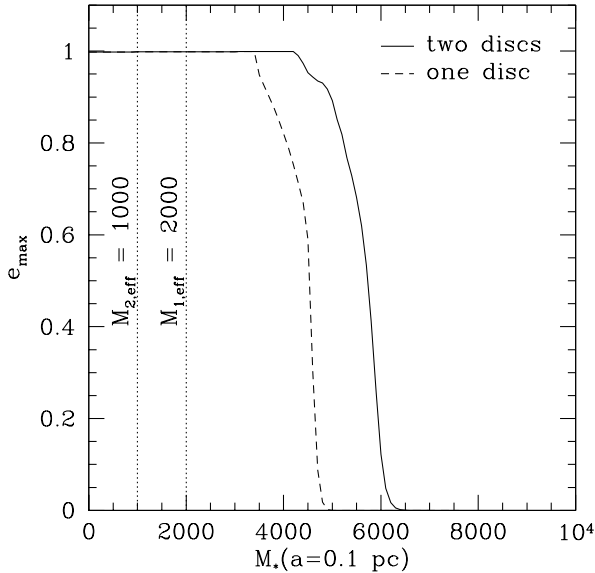
Figure 3. Maximum eccentricity as a function of stellar mass enclosed for $a = 0.1\,\mathrm{pc}$ for one disc (dashed line, $M_{1,\mathrm{eff}} = 2 \times 10^3\mathrm{M_\odot}$) and two discs (solid line, $M_{1,\mathrm{eff}} = 2 \times 10^3\mathrm{M_\odot}$ and $M_{2,\mathrm{eff}} = 10^3\mathrm{M_\odot}$). Also shown is $M_{1,\mathrm{eff}}$ and $M_{2,\mathrm{eff}}$ for the comparison of scales. For sufficiently large stellar bulges ($M_*(a = 0.1\,\mathrm{pc}) \gtrsim 3500\mathrm{M_\odot}$ for one disc and $M_*(a = 0.1\,\mathrm{pc}) \gtrsim 4500\mathrm{M_\odot}$ for two discs), large eccentricities cannot be achieved and the Kozai mechanism is suppressed.

tions are suppressed for initially low eccentricity orbits for a sufficiently massive spherical stellar distribution. Similarly, Karas & Šubr (2007) also showed that a spherical stellar distribution whose mass is comparable to the perturbing disc mass will suppress Kozai oscillations for initially circular orbits (see the third panel of their Figure 2). The results of this paper are in broad agreement with these previous results and point out that inclusion of additional rings, which breaks axial symmetry, do not change the basic results for small *initial* eccentricities.

4   DISCUSSION AND CONCLUSION

I find that the apsidal precession induced by the stellar bulge (using a realistic estimate for its mass) greatly reduces the impact of the Kozai mechanism in the GC. Hence, the Kozai oscillations central to LBK's elegant mechanism is likely not due to the stellar discs. The calculation in LBK is more realistic in that it captures the dynamics of the system without resorting to the perturbative scheme used in the present study. However, this study is complementary because it outlines the regions of parameter space where secular effects are important.

According to the present work, the young stellar disc is not likely to induce Kozai oscillations, but other nonspherical components to the potential may able to do so if they are sufficiently strong. One possibility is a significantly flatten (of order unity) stellar bulge. A detailed study of the degree of flattening required to induce Kozai oscillation is interesting, but it is beyond the scope of this work. On the-

oretical grounds, such a significantly flattened bulge may be unlikely because resonant relaxation (Rauch & Tremaine 1996) would isotropise the bulge stars (Levin 2007, see for instance,) over their 10 Gyr lifetime. Observationally, there is no evidence for a flatten bulge as the velocity distribution of the late-type stars appear to be consistent with isotropy (Genzel et al. 1996).

The Kozai mechanism may be more applicable in other galactic nuclei such as M31, which has a massive stellar disk (the P1/P2 disk). In M31, the P1/P2 disc (Tremaine 1995; Peiris & Tremaine 2003; Chang et al. 2007) is $\sim 10\%$ of the mass of the $1.4 \times 10^8\mathrm{M_\odot}$ SMBH (Bender et al. 2005), whereas the stellar bulge potential is much smaller, i.e., $M_* < 10^6\mathrm{M_\odot}$ at 1" or ≈ 4 pc (Peiris & Tremaine 2003). Chang et al. (2007) has suggested that the nonaxisymmetric potential of the P1/P2 disc modifies gas orbits such that they are confined to be inside of 1 pc around the SMBH. High inclination stellar orbits may also undergo Kozai oscillations in this case. The implications for these stars undergoing Kozai oscillations in the nucleus of M31 would be an interesting topic for further study.

REFERENCES

Alexander T., 2005, Phys. Rep., 419, 65
Bender R., Kormendy J., Bower G., Green R., Thomas J., Danks A. C., Gull T., Hutchings J. B., Joseph C. L., Kaiser M. E., Lauer T. R., Nelson C. H., Richstone D., Weistrop D., Woodgate B., 2005, ApJ, 631, 280
Berukoff S. J., Hansen B. M. S., 2006, ApJ, 650, 901
Brouwer D., Clemence G. M., 1961, Methods of celestial mechanics. New York: Academic Press, 1961
Chang P., 2008, ApJ, 684, 236
Chang P., Murray-Clay R., Chiang E., Quataert E., 2007, ApJ, 668, 236
Eckart A., Genzel R., 1997, MNRAS, 284, 576
Eggleton P. P., Kiseleva L. G., Hut P., 1998, ApJ, 499, 853
Eggleton P. P., Kiseleva-Eggleton L., 2001, ApJ, 562, 1012
Fabrycky D., Tremaine S., 2007, ApJ, 669, 1298
Ford E. B., Kozinsky B., Rasio F. A., 2000, ApJ, 535, 385
Genzel R., Schödel R., Ott T., Eisenhauer F., Hofmann R., Lehnert M., Eckart A., Alexander T., Sternberg A., Lenzen R., Clénet Y., Lacombe F., Rouan D., Renzini A., Tacconi-Garman L. E., 2003, ApJ, 594, 812
Genzel R., Thatte N., Krabbe A., Kroker H., Tacconi-Garman L. E., 1996, ApJ, 472, 153
Gerhard O., 2001, ApJL, 546, L39
Ghez A. M., Hornstein S. D., Lu J. R., Bouchez A., Le Mignant D., van Dam M. A., Wizinowich P., Matthews K., Morris M., Becklin E. E., Campbell R. D., Chin J. C. Y.,

Hartman S. K., Johansson E. M., Lafon R. E., Stomski P. J., Summers D. M., 2005, ApJ, 635, 1087

Goldreich P., Tremaine S. D., 1978, Icarus, 34, 240

Goodman J., 2003, MNRAS, 339, 937

Gould A., Quillen A. C., 2003, ApJ, 592, 935

Hansen B. M. S., Milosavljević M., 2003, ApJL, 593, L77

Hills J. G., 1988, Nature, 331, 687

Holman M., Touma J., Tremaine S., 1997, Nature, 386, 254

Innanen K. A., Zheng J. Q., Mikkola S., Valtonen M. J., 1997, AJ, 113, 1915

Ivanov P. B., Polnarev A. G., Saha P., 2005, MNRAS, 358, 1361

Karas V., Šubr L., 2007, A&A, 470, 11

Kiseleva L. G., Eggleton P. P., Mikkola S., 1998, MNRAS, 300, 292

Kolychalov P. I., Syunyaev R. A., 1980, Pis ma Astronomicheskii Zhurnal, 6, 680

Kozai Y., 1962, AJ, 67, 591

Levin Y., 2007, MNRAS, 374, 515

Levin Y., Beloborodov A. M., 2003, ApJL, 590, L33

Lidov M. L., 1962, P&SS, 9, 719

Löckmann U., Baumgardt H., 2008, MNRAS, 384, 323

Löckmann U., Baumgardt H., Kroupa P., 2008, ApJL, 683, L151

Lu J. R., Ghez A. M., Hornstein S. D., Morris M., Matthews K., Thompson D. J., Becklin E. E., 2006, Journal of Physics Conference Series, 54, 279

Murray C. D., Dermott S. F., 2000, Solar System Dynamics. Cambridge University Press, Cambridge

Nayakshin S., Cuadra J., 2005, A&A, 437, 437

Nayakshin S., Cuadra J., Springel V., 2007, MNRAS, 379, 21

Nayakshin S., Dehnen W., Cuadra J., Genzel R., 2006, MNRAS, 366, 1410

Paczynski B., 1978, Acta Astronomica, 28, 91

Paumard T., Genzel R., Martins F., Nayakshin S., Beloborodov A. M., Levin Y., Trippe S., Eisenhauer F., Ott T., Gillessen S., Abuter R., Cuadra J., Alexander T., Sternberg A., 2006, ApJ, 643, 1011

Peiris H. V., Tremaine S., 2003, ApJ, 599, 237

Perets H. B., Hopman C., Alexander T., 2007, ApJ, 656, 709

Press W. H., Teukolsky S. A., Vettering W. T., Flannery B. P., 1992, Numerical Recipes. Cambridge Univ. Press, Cambridge

Rauch K. P., Tremaine S., 1996, New Astronomy, 1, 149

Schödel R., Eckart A., Alexander T., Merritt D., Genzel R., Sternberg A., Meyer L., Kul F., Moultaka J., Ott T., Straubmeier C., 2007, A&A, 469, 125

Thompson T. A., Quataert E., Murray N., 2005, ApJ, 630, 167

Tremaine S., 1995, AJ, 110, 628

Tremaine S., Zakamska N. L., 2004, in Holt S. S., Deming D., eds, The Search for Other Worlds Vol. 713 of American Institute of Physics Conference Series, Extrasolar Planet Orbits and Eccentricities. pp 243–252

Šubr L., Karas V., 2005, A&A, 433, 405

Šubr L., Karas V., Huré J.-M., 2004, MNRAS, 354, 1177

Yu Q., Lu Y., Lin D. N. C., 2007, ApJ, 666, 919